



Fig. 1 Plot of flight-test and predicted values of  $T$ .

regression of data from flight tests was performed. The data used in this regression are shown in Table 1. Fifty passes using two different aircraft were made to ensure a representative database. Roll angle, speed, and time were obtained from flight data recorders aboard the aircraft. The time  $T$  to roll wings level was obtained by subtracting the time at which  $\phi$  began to decrease monotonically from the time at which  $\phi$  was 5 deg or less (thus, pilot and aircraft reaction times are not included in  $T$ ). The results of the linear regression are  $A = 0.8$ ,  $B = 5.0$  when  $\phi_0$  has units of degrees and  $V$  has units of ft/s. Thus, the time  $T$  is given approximately by

$$T = 5\phi_0/V + 0.8 \quad (4)$$

The correlation coefficient for the linear fit is 0.93, which indicates a very strong linear correlation.

Values of  $T$  obtained from Eq. (4) are shown in Table 1 alongside the corresponding flight-test values. The data in Table 1 indicate that all of the least-squares values of  $T$  are within 0.4 s of the flight-test values. However, 90% of the least-squares values of  $T$  are within 0.3 s of the corresponding flight-test values. Furthermore, 72% and 50% of the least-squares values of  $T$  are within 0.2 s and 0.1 s, respectively, of the corresponding flight-test values. Thus, the data indicate that the linear fit described earlier accurately represents the flight-test data.

Graphical representations of the flight-test data and the line of best fit are shown in Fig. 1. It is again evident that, for most values of  $\phi_0/V$ , the linear fit is accurate. However, for small values of  $\phi_0/V$ , the linear fit appears to be conservative. This is to be expected, since a fundamental assumption made earlier is that the time  $T$  to roll wings level is large enough so that the quantity  $\exp(-kT)$  may be neglected. For small  $\phi_0/V$ , this condition is clearly not satisfied. A simple solution to this problem is to use a different linear approximation for  $\phi_0/V$  small, say,  $\phi_0/V < 0.05$ . A reasonable linear equation for this case is the equation that joins (0, 0) and (0.05, 1.0), namely,

$$T = 21\phi_0/V$$

since the flight-test time to roll wings level is about 1.0 s when  $\phi_0/V = 0.05$ . This equation is plotted as the dashed line in Fig. 1. Thus, the complete algorithm for approximating the time  $T$  to roll wings level in the A-7E is

$$T = \begin{cases} 5\phi_0/V + 0.8 & \text{for } \phi_0/V \geq 0.05 \\ 21\phi_0/V & \text{for } \phi_0/V < 0.05 \end{cases}$$

where, again,  $\phi_0$  is the bank angle in degrees and  $V$  is the airspeed in ft/s.

## Reference

<sup>1</sup>Perkins, C. D. and Hage, R. E., *Airplane Performance Stability and Control*, Wiley, New York, 1949, pp. 433-436.

## Estimation of the Parameters of Convection Dynamics

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### Nomenclature

$a_0, b_0$	= constants, $a_0 \cong 0.12$ , $b_0 \cong 0.88$
$C_i$	= fraction of cloud cover of type $i$
$C_p$	= specific heat at constant pressure for air
$E_0$	= surface evaporative flux
$F_M$	= wind profile function
$F_H$	= potential temperature profile function
$F_L\downarrow, F_L\uparrow$	= downward and upward long-wave radiative fluxes, respectively
$F_S\downarrow, F_S\uparrow$	= downward and upward short-wave radiative fluxes, respectively
$G_0$	= energy flux into the ground
$g$	= acceleration due to gravity
$H_0$	= surface heat flux
$h$	= height of the convective boundary layer
$I_0$	= solar constant adjusted for variation of orbit radius
$k$	= von Kármán constant, $\cong 0.41$
$k_1$	= constant, $\cong 0.1$
$L$	= Obukhov length, $L \equiv -u_*^3/(kg/\theta)(H_0/\rho C_p)(1 + 0.61C_p T/\lambda\beta)$
$m$	= absorption of radiation by water vapor, $m \cong 0.18$
$P$	= pressure
$R$	= gas constant for dry air
$R_n$	= net radiation
$T, T_g$	= air temperature, ground temperature
$U$	= horizontal wind speed
$u_*$	= friction velocity, $\equiv (\tau_0/\rho)^{1/2}$
$w_*$	= convective velocity, $\equiv u_*(-h/kL)^{1/3}$
$z, z_A, z_S$	= height, height of anemometer, height of thermometer
$z_0, z_H$	= roughness lengths for momentum and heat
$\alpha_1$	= constant, $\cong 0.17$
$\alpha_A, \alpha_i, \alpha_g$	= albedo of air, of cloud of type $i$ , and of ground
$\beta$	= Bowen ratio, $\equiv H_0/\lambda E_0$
$\gamma$	= ratio of ground flux to net radiation
$\epsilon_g$	= emissivity of the ground, $\epsilon_g \cong 0.9$
$\zeta$	= zenith angle
$\theta, \theta_0$	= potential temperature at heights $z$ and $z_H$
$\theta_*$	= temperature scaling parameter, $\equiv -H_0/\rho C_p u_*$
$\lambda$	= latent heat of vaporization
$\xi$	= dummy variable
$\rho$	= air density
$\sigma$	= Stefan-Boltzmann constant, $\sigma \cong 5.687 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$
$\tau_0$	= surface stress
$\phi_M, \phi_H$	= nondimensional wind shear and potential temperature gradient
$\psi_M, \psi_H$	= buoyancy correction terms to wind and potential temperature profiles

### Introduction

THE important role that each of the parameters  $w_*$ ,  $h$ , and  $-h/L$  plays in convection dynamics and gust modeling in the boundary layer is described in a companion paper.<sup>1</sup> The

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parameters  $w_*$  and  $L$  depend on surface fluxes—they are not measured directly. Although the height  $h$  can be measured, e.g., by acoustic sounders or lidars, usually these measurements are not available. In this Note, we indicate how the parameters  $w_*$ ,  $h$ , and  $-h/L$  can be estimated from routine meteorological observations.

### Method of Estimation

We start with the surface energy budget,

$$H_0 + \lambda E_0 = R_n - G_0 \quad (1)$$

Equation (1) can be simplified by parameterizing  $\lambda E_0$  and  $G_0$  as<sup>2</sup>

$$\lambda E_0 = H_0/\beta \quad (2)$$

$$G_0 = \gamma R_n \quad (3)$$

Typical climatological values of  $\beta$  for our applications are 0.3–2.5, based on measurements from the Wangara<sup>3</sup> and Koorin<sup>4</sup> experiments (although values over the desert can reach as high as 10). Measurements from the lysimeter at Aspendale, Australia, indicate that during the daytime  $\gamma \sim 0.1$ –0.35; Slatyer and McIlroy<sup>5</sup> use  $\gamma = 0.25$  in their calculations and obtained good results. We adopt this value for  $\gamma$ .

We now seek a solution to Eqs. (1–3) by rewriting  $R_n$  in terms of the quantities that we can determine from our observations. By definition, the net radiation  $R_n$  can be written as

$$R_n = F_s \downarrow - F_s \uparrow + F_L \downarrow - F_L \uparrow \quad (4)$$

Paltridge and Platt<sup>6</sup> suggest that  $F_s \downarrow$  can be expressed by

$$F_s \downarrow = I_0 \left( 1 - \sum_i \alpha_i C_i \right) \left[ 1 - m - \alpha_A \left( 1 - \sum_i C_i \right) \right] \cos \zeta \quad (5)$$

where  $C_i$  is the cloud fraction (divided into the categories of low stratiform, low cumulus, altocumulus and altostratus, and cirrus) and  $\alpha_i$  the cloud global albedo (0.50, 0.60, 0.55, and 0.35 for the four categories, respectively). The planetary albedo  $\alpha_A$  due to Rayleigh scattering is given by

$$\alpha_A \cong 0.28/(1 + 6.43 \cos \zeta) \quad (6)$$

The surface albedo can be written as<sup>6</sup>

$$\alpha_g(\zeta) = \alpha_1 + (1 - \alpha_1) \exp[-(180k_1/\pi)(\pi/2 - \zeta)] \quad (7)$$

where  $\alpha_1 \cong 0.17$  and  $k_1 \cong 0.1$  for our applications (i.e., for a surface between a eucalyptus forest and dry grassland). For diffuse radiation, assuming isotropic scattering, the surface

albedo is given by

$$\begin{aligned} \alpha_g(\text{diff}) \cong & 2 \int_0^{\pi/2} \alpha_g(\zeta) \sin \zeta \cos \zeta \, d\zeta = (\alpha_1 \sin^2 \zeta) \pi_0^{1/2} \\ & + \left\{ \left( \frac{1 - \alpha_1}{4 + k_1^2} \right) \exp \left[ \left( \frac{-180k_1}{\pi} \right) \left( \frac{\pi}{2} - \zeta \right) \right] \right. \\ & \times (k_1 \sin 2\zeta - 2 \cos 2\zeta) \left. \right\}_{\pi_0^{1/2}} \cong 0.584 \end{aligned} \quad (8)$$

The global upward short-wave radiation is

$$F_s \uparrow = \bar{\alpha}_g F_s \downarrow \quad (9)$$

where  $\bar{\alpha}_g$  is the weighted average of Eqs. (7) and (8),

$$\bar{\alpha}_g = a_0 \alpha_g(\zeta) + b_0 \alpha_g(\text{diff}) \quad (10)$$

and  $a_0 \cong 0.88$  and  $b_0 \cong 0.12$  from the measurements of Collins.<sup>7</sup>

Approximately 0.7 of the total long-wave radiation is emitted outside of the 8–14  $\mu\text{m}$  window. The net long-wave radiation for these wavelengths is always small and, to a first approximation, can be neglected compared to the transfer within the window region. Clouds contribute about  $60 \sum C_i$  ( $\text{Wm}^{-2}$ ) to the downward long-wave flux<sup>6</sup>; hence,

$$F_L \downarrow - F_L \uparrow = 60 \sum_i C_i - 0.27 \sigma \bar{T}_g^4 \quad (11)$$

To proceed further, we employ the equations for the vertical profiles of wind and potential temperature in the surface layer. The wind is measured at height  $z_A = 10$  m. The roughness length for momentum is denoted by  $z_0$ . On the other hand, the potential temperature is derived from measurements of the temperature at the screen height  $z_s = 1.5$  m. The equivalent length to  $z_0$  for the temperature profile is denoted by  $z_H$ , but its value is less well known than  $z_0$ . We adopt a typical value<sup>8</sup> of  $z_0/z_H = 10$ . The profile equations are

$$\frac{k\bar{U}}{u_*} = \left[ \ell n \left( \frac{z_A}{z_0} \right) - \psi_M \left( \frac{z_A}{L} \right) + \psi_M \left( \frac{z_0}{L} \right) \right] = F_M \quad (12)$$

$$\frac{k(\bar{\theta} - \bar{\theta}_0)}{\theta_*} = \left[ \ell n \left( \frac{z_s}{z_H} \right) - \psi_H \left( \frac{z_s}{L} \right) + \psi_H \left( \frac{z_H}{L} \right) \right] = F_H \quad (13)$$

The stability functions  $\psi_M$  and  $\psi_H$  for the unstable conditions pertaining to this study are given by<sup>9</sup>

$$\psi_M(\xi) = 2\ell n[(1 + \phi_M^{-1})/2] + \ell n[(1 + \phi_M^{-2})/2] - 2 \tan^{-1} \phi_M^{-1} \quad (14)$$

$$\psi_H(\xi) = 2\ell n[(1 + \phi_H^{-1})/2] \quad (15)$$

Table 1 Input parameters for calculation of surface fluxes

Location	Date	Local time	$\bar{U}_s$ , $\text{m} \cdot \text{s}^{-1}$	$\bar{T}_s$ , $^{\circ}\text{C}$	$P$ , kPa	$z_0$ , m	$h$ , m	$\beta$	$C_1$	$C_2$	$C_3$	$C_4$
Canberra	25/1/83	1410	2.0	23.6	952	0.50	2000	0.50	0.00	0.50	0.00	0.00
Redcliffe	26/1/83	0940	2.0	27.0	1020	0.25	1600	0.75	0.00	0.00	0.00	0.00
Renmark	21/2/83	1520	2.5	35.0	1009	0.25	2400	1.00	0.00	0.00	0.00	0.00
Bourke	26/2/83	1800	2.5	34.0	1011	0.15	2000	1.50	0.00	0.25	0.00	0.00
Witchellina	27/2/83	1810	2.5	40.0	995	0.05	3000	2.00	0.00	0.00	0.00	0.38
Hebel	29/5/83	1530	2.0	17.5	1004	0.05	1400	0.50	0.00	0.00	0.00	0.00
Wiluna	20/7/83	1640	2.0	20.0	963	0.05	1100	2.00	0.00	0.00	0.00	0.00
Biluela	1/11/83	1438	5.0	28.0	990	0.10	2000	0.75	0.00	0.50	0.00	0.00
Griffith	22/2/84	1525	2.5	30.0	1016	0.05	2000	1.00	0.00	0.00	0.00	0.00
Boggabri	2/3/84	1000	2.5	27.0	1000	0.10	1800	0.75	0.00	0.00	0.00	0.00
Wallacia	4/11/84	1130	2.5	20.0	1012	0.50	1200	0.50	0.00	0.88	0.00	0.00
Hoxton Park	3/2/85	1005	2.0	24.0	1020	0.50	1300	1.00	0.00	0.00	0.00	0.50
Cape Keer Weer	27/6/85	0930	2.5	25.0	1023	0.002	2000	1.00	0.00	0.00	0.00	0.00

Table 2 Summary of results of calculations of fluxes

Location	$L$ , m	$u_*$ , $\text{m} \cdot \text{s}^{-1}$	$w_*$ , $\text{m} \cdot \text{s}^{-1}$	$H_0$ , $\text{W} \cdot \text{m}^{-2}$	$\lambda E_0$ , $\text{W} \cdot \text{m}^{-2}$	$R_n$ , $\text{W} \cdot \text{m}^{-2}$	$G_0$ , $\text{W} \cdot \text{m}^{-2}$
Canberra	-25.0	0.35	2.01	121.4	242.7	485.5	121.4
Redcliffe	-11.7	0.30	2.09	187.1	249.5	582.1	145.5
Renmark	-16.9	0.36	2.50	216.5	216.5	577.4	144.4
Bourke	-29.8	0.29	1.56	65.2	43.5	145.0	36.2
Witchellina	-11.8	0.24	2.04	96.8	48.4	193.7	48.4
Hebel	-17.6	0.18	1.06	26.9	53.7	107.5	26.9
Wiluna	-6.1	0.21	1.59	121.1	60.6	242.3	60.6
Biloela	-79.2	0.48	1.89	109.2	145.6	339.7	84.9
Griffith	-6.8	0.26	2.30	203.1	203.1	541.7	135.4
Boggabri	-13.8	0.28	1.91	124.3	165.8	386.8	96.7
Wallacia	-58.0	0.39	1.45	79.8	159.6	319.2	79.8
Hoxton Park	-22.9	0.35	1.82	155.1	155.1	413.6	103.4
Cape Keer Weer	-3.1	0.15	1.77	92.8	92.8	247.4	61.8

where  $\phi_M = (1 - 16\xi)^{-\frac{1}{4}}$ ,  $\phi_H = (1 - 16\xi)^{-\frac{1}{2}}$ , and  $\xi = z_A/L$ ,  $z_0/L$ ,  $z_s/L$ , or  $z_H/L$ .

We approximate the ground temperature by the temperature at  $z_H$ ,

$$\bar{T}_g \cong T(z_H) = \bar{\theta}_0 (P/1000)^{R/C_p} \quad (16)$$

From Eqs. (1-13) and (16), we obtain an equation for  $H_0$  as

$$H_0 = \left\{ I_0 (1 - \bar{\alpha}_g) \left( 1 - \sum_i \alpha_i C_i \right) \left[ 1 - m - \alpha_A \left( 1 - \sum_i C_i \right) \right] \cos \zeta \right. \\ \left. + 60 \sum_i C_i - 0.27 \sigma [\bar{\theta} + (H_0 / \rho C_p k^2 \bar{U}) F_M F_H]^4 \right. \\ \left. \times (P/1000)^{4R/C_p} \right\} (1 - \gamma) \beta / (1 + \beta) \quad (17)$$

Equation (17) is implicit in  $H_0$  and the solution requires iteration. Initial values of  $H_0$  and  $u_*$  are assumed. The Obukhov length  $L$  is the calculated and the functions  $F_M$  and  $F_H$  are found from Eqs. (12-15). A new estimate of  $H_0$  is then obtained from Eq. (17) and a new value of  $u_*$  from Eq. (12). Having obtained values for  $H_0$  and  $u_*$ , we are in position to obtain a new estimate of  $L$ . The functions  $F_M$  and  $F_H$  are then re-evaluated, using the new value of  $L$  and the cycle continues until  $L$  is known to 0.01%.

Wind, temperature, pressure, and cloud data are obtained from routine surface observations. The roughness length  $z_0$  is estimated from the vegetative cover and the Bowen ratio  $\beta$  from rainfall information. The height of the convective boundary layer is determined from the intersection of

boundary-layer potential temperature and the radiosonde temperature profile. Table 1 summarizes the input parameters for several case studies, and Table 2 gives the results of the calculations for these case studies.

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## Notice to Subscribers

We apologize that this issue was mailed to you late. As you may know, AIAA recently relocated its headquarters staff from New York, N.Y. to Washington, D.C., and this has caused some unavoidable disruption of staff operations. We will be able to make up some of the lost time each month and should be back to our normal schedule, with larger issues, in just a few months. In the meanwhile, we appreciate your patience.